75. **(B)** (p. 480) ***English/Rhetorical Skills/Organization/Paragraph-Level Structure.*** The function of the first paragraph is to place Aristotle in a certain context.

Section 2: Mathematics

1. **(A)** (p. 482) ***Mathematics/Algebra/Solving Algebraic Equations or Inequalities with One Variable/Equations Involving Rational Expressions.*** Solve for x: $\frac{1}{x}+\frac{1}{x}=8 \Rightarrow \frac{2}{x}=8 \Rightarrow x=\frac{1}{4}$. Also, one can reason that $\frac{1}{x}$ and $\frac{1}{x}$ are equal, and since their sum is 8, $\frac{1}{x}$ equals 4. Thus, $x=\frac{1}{4}$.

2. **(K)** (p. 482) ***Mathematics/Algebra/Expressing and Evaluating Algebraic Functions/Function Notation.*** $3x-4y=3(2)-4(-1)=6+4=10$.

3. **(C)** (p. 483) ***Mathematics/Arithmetic/Common Arithmetic Items/Percents.*** 20% of 600 boys equals $0.20(600)=120$ boys on the honor roll. 30% of 400 girls equals $0.30(400)=120$ girls on the honor roll. Therefore, there are 120 boys + 120 girls = 240 students on honor roll.

4. **(K)** (p. 483) ***Mathematics/Arithmetic/Common Arithmetic Items/Properties of Numbers.*** Since an even number times any other whole number yields an even number, the correct answer is (K). Since the variable t is outside the brackets and parentheses, it must be multiplied by everything within the brackets and parentheses. Therefore, t must be even. None of the other letters guarantees an even result.

   Alternatively, for each letter, assume that that letter only is even and that all other numbers are odd. Only t generates an even result under those circumstances.

5. **(E)** (p. 483) ***Mathematics/Statistics and Probability/Data Representation/Tables (Matrices).*** The trick of this question is to see that the number of flies in each successive week is four times the number of the previous week. The final count should be $4 \cdot 192 = 768$.

6. **(H)** (p. 484) ***Mathematics/Arithmetic/Simple Manipulations*** and ***Statistics and Probability.*** Use the formula for finding the number of permutations: $3! = 3 \cdot 2 \cdot 1 = 6$.

   Alternatively, simply count the number of possibilities: *ABC, ACB, BAC, BCA, CAB, CBA.*

7. **(D)** (p. 484) ***Mathematics/Coordinate Geometry/The Coordinate System.*** Since the x-coordinate of both points is 2, the line runs parallel to the y-axis. The x-coordinate of the midpoint will also be 2. As for the y-coordinate, the midpoint is halfway between 2 and –2: 0.

8. **(H)** (p. 484) ***Mathematics/Algebra/Solving Algebraic Equations or Inequalities with One Variable/Equations Involving Absolute Value.*** Since the absolute value of xy is positive, xy itself must be positive (since $|xy| = xy$). Therefore, both x and y have the same sign. They might both be positive, or they might both be negative. (F), (G), (J), and (K) can all be true; x and y cannot, however, have different signs because a positive times a negative yields a negative result.

   Alternatively, substitute some numbers. If $x > 0 > y$, then x could be 1 and y could be -1, and $(1)(-1) = -1$.

9. **(D)** (p. 485) ***Mathematics/Geometry/Rectangles and Squares.*** Convert the dimensions shown to real dimensions. Since 1 centimeter is equal to 4 meters, the width of the room is 4 meters, and the length is 4.8. Thus, the area of the room is $4 \cdot 4.8 = 19.2$.

10. **(G)** (p. 485) ***Mathematics/Algebra/Manipulating Algebraic Expressions/Basic Algebraic Manipulations.*** This question tests power rules. $30,000 \times 20 = 600,000 = 6 \times 10^5$. (One power of 10 for each zero.)

11. **(B)** (p. 485) ***Mathematics/Arithmetic/Solving Complicated Arithmetic Application Items*** and ***Algebra/Solving Simultaneous Equations.*** Use simultaneous equations to solve this problem. If x is the quantity of chocolates and y is the quantity of caramels, then: $x + y = 4$ and $3x + 2y = 10 \Rightarrow y = 4 - x \Rightarrow 3x + 2(4 - x) = 10 \Rightarrow 3x + 8 - 2x = 10 \Rightarrow x = 10 - 8 = 2$.

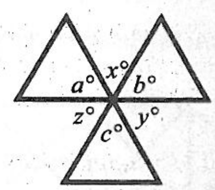
    Alternatively, test the answer choices, starting with (C). If Karen buys 2.5 pounds of chocolates, she bought $4 - 2.5 = 1.5$ pounds of caramels and the total cost is $(2.5 \cdot 3) + (1.5 \cdot 2) = 7.50 + 3 = \$10.50$. This is too much money. Since chocolates are more expensive than caramels, Karen bought less than 2.5 pounds of chocolates. Try (B): 2 pounds of chocolates and 2 pounds of caramels cost $(2 \cdot 3) + (2 \cdot 2) = 6 + 4 = 10$.

12. **(F)** (p. 485) ***Mathematics/Statistics and Probability/Averages.*** All three numbers total $3 \cdot 80 = 240$. The total of the two given numbers is $2 \cdot 77 = 154$. The missing number is $240 - 154 = 86$.

13. **(C)** (p. 486) ***Mathematics/Arithmetic/Common Arithmetic Items/Ratios.*** The total number of ratio parts in the ratio $5:2$ is 7, and 10 is not evenly divisible by 7. As for (A), (C), (D), and (E), these are incorrect because 10 is evenly divisible by the total number of ratio parts.

14. **(G)** (p. 486) ***Mathematics/Algebra/Solving Algebraic Equations or Inequalities with One Variable/Simple Equations.*** Treat the equation as a proportion: $\frac{4}{5} = \frac{x}{4} \Rightarrow 4(4) = 5x \Rightarrow x = \frac{16}{5}$.

15. **(D)** (p. 486) ***Mathematics/Geometry/Lines and Angles*** and ***Triangles/Working with Triangles.*** Label the unlabeled angles:



    Since the measure of the degrees in a circle is 360, the sum of x, y, and z plus the sum of a, b, and c is 360. What is the value of the angles inside the triangles? Since they are equilateral triangles, each angle is 60°: $3(60) + x + y + z = 360 \Rightarrow x + y + z = 180$.

    Alternatively, since you can determine from the given figure that x, y, and z each measure 60 and therefore total 180, you can use alternate interior angles to solve this item. After labeling the unlabeled angles, as done above, simply recognize that $a° = y°$, $b° = z°$, and $c° = x°$. So, $x + y + z = a + b + c = 180$.

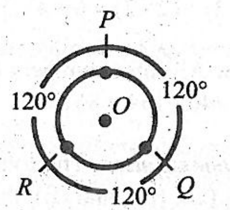
16. **(H)** (p. 487) ***Mathematics/Arithmetic/Solving Complicated Arithmetic Application Items.*** If Peter spent $\frac{1}{4}$ of his allowance on Monday, he had $\frac{3}{4}$ of his allowance left. Then, he spent $\frac{1}{3}$ of that $\frac{3}{4}$ on Tuesday: $\frac{1}{3} \cdot \frac{3}{4} = \frac{1}{4}$. After spending the additional $\frac{1}{4}$, he was left with $\frac{3}{4} - \frac{1}{4} = \frac{1}{2}$ of the original allowance. Substitution of numbers would also work, but the arithmetic would be the same.

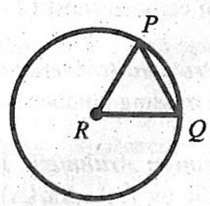
17. **(B)** (p. 487) ***Mathematics/Arithmetic/Common Arithmetic Items/Proportions and Direct-Inverse Variation.*** There are three ways to solve the problem. The simplest and most direct is to reason that if 100 bricks weigh p

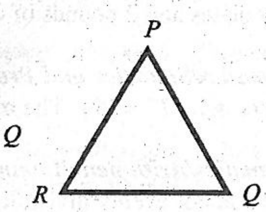
pounds, 20 bricks, which is $\frac{1}{5}$ of 100, must weigh $\frac{1}{5}$ of p. This same reasoning can be expressed using a direct proportion. The fewer the bricks, the lesser the weight, so: $\frac{x}{p}=\frac{20}{100} \Rightarrow 100x = 20p \Rightarrow x=\frac{20p}{100} \Rightarrow x=\frac{p}{5}$.

Alternatively, substitute some numbers. Assume that 100 bricks weigh 100 pounds, which is 1 pound each. 20 bricks weigh 20 pounds. On the assumption that $p=100$, the correct formula will generate the number 20.

18. **(K)** (p. 488) ***Mathematics/Geometry/Circles*** and ***Triangles/Working with Triangles.*** The following drawings show that (I), (II), and (III) are possible.







19. **(A)** (p. 488) ***Mathematics/Statistics and Probability/Data Representation/Tables (Matrices)*** and ***Arithmetic/ Common Arithmetic Items/Percents.*** This problem can be solved with the "change-over" principle, but that would require five different calculations. It is always easier and faster to find the greatest ratio of the increased value to the original value. Therefore, look at the successive ratios. The price doubles during the first 5-year period. However, it less than doubles during each of the other periods. Thus, the answer is (A).

20. **(F)** (p. 488) ***Mathematics/Algebra/Manipulating Algebraic Expressions/Factoring Expressions.*** The easiest approach is just to perform the multiplication for the answer choices:

F. $(x-2)(x+6)=x^2+4x-12$ ✓
G. $(x-4)(x+3)=x^2-x-12$ ✗
H. $(x-6)(x+2)=x^2-4x-12$ ✗
J. $(x+2)(x+6)=x^2+8x+12$ ✗
K. $(x+3)(x+4)=x^2+7x+12$ ✗

21. **(D)** (p. 489) ***Mathematics/Algebra/Solving Algebraic Equations or Inequalities with One Variable/Simple Equations.*** Since the average of $3x-2$ and $2x-3$ is 10, the sum is 20: $3x-2+2x-3=20 \Rightarrow 5x-5=20 \Rightarrow 5x=25 \Rightarrow x=5$. One package weighs $3(5)-2=13$ pounds and the other package weighs $2(5)-3=7$ pounds. The weight difference is $13-7=6$ pounds.

22. **(G)** (p. 489) ***Mathematics/Statistics and Probability/Averages.*** This question is a variation on the theme of an average with missing elements. Since 10 students have scores of 75 or more, the total of their scores is at minimum $10 \bullet 75 = 750$. Then, even assuming the other 5 students each scored zero, the average for the 15 would be at least $750 \div 15 = 50$.

23. **(C)** (p. 489) ***Mathematics/Algebra/Manipulating Algebraic Expressions/Manipulating Expressions Involving Exponents.*** Since $16=4^2$, $16^x=\left(4^2\right)^x=4^{2x}$. This problem is also solvable by assuming a value for x. If $x=1$, $16^x=16^1=16$. The correct answer choice will yield the value 16 when 1 is substituted for x:

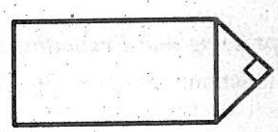
A. $1^{16}=1$ ✗
B. $2^{3(1)}=2^3=8$ ✗

C. $4^{2(1)} = 4^2 = 16$ ✓
D. $8^{2(1)} = 8^2 = 64$ ✗
E. $8^{4(1)} = 8^4 = 4{,}096$ ✗

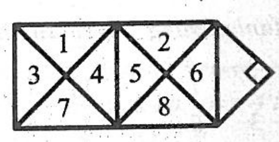
24. (F) (p. 489) ***Mathematics/Geometry/Rectangles and Squares.*** The figure is a square, so the two sides are equal: $2x+1 = x+4 \Rightarrow x = 3$. One side is $x+4 = 3+4 = 7$. Since all four sides are equal, the perimeter is $4 \cdot 7 = 28$.

25. (E) (p. 490) ***Mathematics/Geometry/Rectangles and Squares*** and ***Triangles/45°-45°-90° Triangles*** and ***Working with Triangles.*** If w is the width of the rectangle, the length is $2w$ and the rectangle has an area of $w \cdot 2w = 2w^2$. Then, w is also the length of the hypotenuse of a 45°-45°-90° triangle. Each of the other two sides forming the right angle (altitude and base) is $\frac{1}{2} \cdot w \cdot \sqrt{2} = \frac{\sqrt{2}w}{2}$. The area of the triangle is $\frac{1}{2} \cdot \text{altitude} \cdot \text{base} = \frac{1}{2} \cdot \frac{\sqrt{2}w}{2} \cdot \frac{\sqrt{2}w}{2} = \frac{1}{2} \cdot \frac{w^2}{4}$. The ratio of the area of the rectangle to that of the triangle is $\frac{2w^2}{\frac{w^2}{4}} = \frac{2}{\frac{1}{4}} = \frac{8}{1}$.

The above explanation is difficult to follow without a diagram, so draw one:

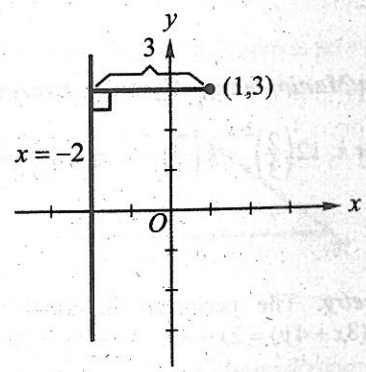


The explanation will not only be easier to follow, but it can be dispensed with altogether. The rectangle is obviously bigger than the triangle, so eliminate (A), (B), and (C). Adding to the figure shows that the area of the triangle is less than $\frac{1}{4}$ of the area of the rectangle. Approximate all of the triangles:



By process of elimination, (E) is correct.

26. (G) (p. 490) ***Mathematics/Coordinate Geometry/The Coordinate System.*** No diagram is provided, so sketch one:



27. (C) (p. 490) ***Mathematics/Arithmetic/Common Arithmetic Items/Proportions and Direct-Inverse Variation.*** Determine how much coffee costs per pound: $\frac{\$12}{5 \text{ pounds}} = \$2.40$ per pound. Therefore, \$30 buys $\frac{\$30}{\$2.40/\text{pound}} = 12.5$ pounds.

Alternatively, the process can be represented in a single direct proportion: $\frac{\text{Cost } X}{\text{Cost } Y} = \frac{\text{Pounds } X}{\text{Pounds } Y} \Rightarrow \frac{\$30}{\$12} = \frac{x}{5} \Rightarrow$ $12x = 150 \Rightarrow x = \frac{150}{12} = 12.5$.

28. (J) (p. 491) ***Mathematics/Geometry/Triangles/Working with Triangles.*** Since the triangles are equilateral, the ratio of their perimeters is the same as the ratio of their sides. Thus, the ratio of their perimeters is also $\frac{3}{12}$, or $\frac{1}{4}$.

Alternatively, find the perimeter of each triangle. Since the triangles are equilateral, the smaller one has a perimeter of $3+3+3=9$, and the larger one has a perimeter of $12+12+12=36$. Therefore, the ratio is: $\frac{9}{36}=\frac{1}{4}$.

29. (E) (p. 491) ***Mathematics/Algebra/Expressing and Evaluating Algebraic Functions/Function Notation.*** Simply substitute -2 for x into the given function: $f(-2) = -3(-2)^3 + 3(-2)^2 - 4(-2) + 8 = -3(-8) + 3(4) - (-8) + 8 = 24 + 12 + 8 + 8 = 52$.

30. (H) (p. 491) ***Mathematics/Arithmetic/Common Arithmetic Items/Percents.*** Add 30 percent to the \$120 wholesale price: $\$120 + (0.30 \bullet \$120) = \$120 + \$36 = \$156$. Then, find the sale price: $\$156 - (0.40 \bullet \$156) = \$156 - \$62.40 = \$93.60$.

31. (A) (p. 492) ***Mathematics/Algebra/Manipulating Algebraic Expressions/Evaluating Expressions.*** $\frac{1}{3}$ of the number equals $\frac{1}{5}$ of the number plus 2: $\frac{1}{3}x = \frac{1}{5}x + 2 \Rightarrow \frac{1}{3}x - \frac{1}{5}x = 2$.

32. (H) (p. 492) ***Mathematics/Geometry/Complex Figures*** and ***Triangles/Working with Triangles*** and ***Rectangles and Squares.*** This is a composite figure. One side of the equilateral triangle is also a side of the square. The triangle has a perimeter of 12, so each side is 4. If the square has a side of 4, then the perimeter is $4+4+4+4=16$.

33. (A) (p. 492) ***Mathematics/Algebra/Manipulating Algebraic Expressions/Evaluating Expressions.*** Substitute $\frac{2}{3}$ for x in the expression and solve for k: $12\left(\frac{2}{3}\right)^2 + k\left(\frac{2}{3}\right) = 6 \Rightarrow 12\left(\frac{4}{9}\right) + k\left(\frac{2}{3}\right) = 6 \Rightarrow 4\left(\frac{4}{3}\right) + k\left(\frac{2}{3}\right) = 6 \Rightarrow \frac{16}{3} + k\left(\frac{2}{3}\right) = 6 \Rightarrow k\left(\frac{2}{3}\right) = \frac{18}{3} - \frac{16}{3} = \frac{2}{3} \Rightarrow k = 1$.

34. (K) (p. 493) ***Mathematics/Geometry.*** The perimeter is equal to the sum of the lengths of the sides: $2(x-2y) + 4(2x+y) = (2x-4y) + (8x+4y) = 2x + 8x - 4y + 4y = 10x$.

Alternatively, substitute some numbers. Assume that $x=3$ and $y=1$. The two short sides are each $3-2(1)=1$, for a total of 2. The four long sides are $2(3)+1=7$, for a total of 28. The perimeter is $28+2=30$. Thus, if $x=3$ and $y=1$, the correct formula will generate the number 30. Only (K) produces the correct value.

35. (D) (p. 493) ***Mathematics/Arithmetic/Solving Complicated Arithmetic Application Items.*** Let x be the number of packages in the van before the first delivery: $\left(x-\frac{2}{5}x\right)-3=\frac{1}{2}x \Rightarrow \frac{3}{5}x-3=\frac{1}{2}x \Rightarrow \frac{3}{5}x-\frac{1}{2}x=3 \Rightarrow \frac{1}{10}x=3 \Rightarrow x=30$.

36. (J) (p. 493) ***Mathematics/Algebra/Manipulating Algebraic Expressions/Evaluating Expressions.*** Perform the indicated operations in the answer choices to determine which one is equal to the expression in the stem $\left(12x^3y^2-8x^2y^3\right)$:

F. $2x^2y^2(4x-y)=8x^3y^2-2x^2y^3$ ✗
G. $4x^2y^2(2xy)=8x^3y^3$ ✗
H. $4x^2y^2(3xy)=12x^3y^3$ ✗
J. $4x^2y^2(3x-2y)=12x^3y^2-8x^2y^3$ ✓
K. $x^3y^3(12xy-8xy)=12x^4y^4-8x^4y^4$ ✗

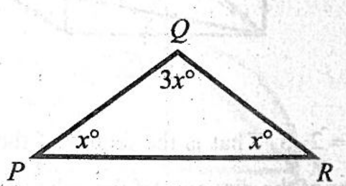
37. (C) (p. 494) ***Mathematics/Algebra/Manipulating Algebraic Expressions/Basic Algebraic Manipulations.*** The fastest way to solve this problem is to simply rewrite the expression: $\frac{1}{1+\frac{1}{x}}=\frac{1}{\frac{x+1}{x}}=1\left(\frac{x}{x+1}\right)=\frac{x}{x+1}$.

Alternatively, substitute numbers into the expression. If $x=1$, then: $\frac{1}{1+\frac{1}{x}}=\frac{1}{1+\frac{1}{1}}=\frac{1}{1+1}=\frac{1}{2}$. If $x=1$, both (B) and (C) generate $\frac{1}{2}$. Therefore, try another number. If $x=2$, the correct answer should generate the value $\frac{2}{3}$; (B) is eliminated and (C) must be correct.

38. (G) (p. 494) ***Mathematics/Arithmetic/Common Arithmetic Items/Percents.*** Use S and T as unknowns. Since S is 150 percent of T, S equals $1.5T$. Substitute $1.5T$ for S: $\frac{T}{1.5T+T}=\frac{T}{2.5T}=\frac{1}{2.5}=40$ percent.

Alternatively, substitute real numbers. Let S be 15 and T be 10; then, $\frac{T}{S+T}=\frac{10}{10+15}=\frac{10}{25}=40\%$.

39. (C) (p. 494) ***Mathematics/Geometry/Lines and Angles.*** No figure is provided, so sketch one:



$x°+x°+3x°=180° \Rightarrow 5x°=180° \Rightarrow x=36$.

40. (G) (p. 495) ***Mathematics/Arithmetic/Common Arithmetic Items/Proportions and Direct-Inverse Variation.*** Use a direct proportion: $\frac{C}{x}=\frac{d}{b} \Rightarrow Cb=dx \Rightarrow C=\frac{dx}{b}$.

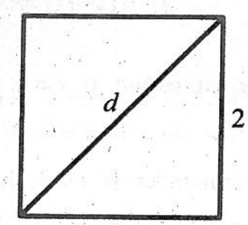
41. (D) (p. 495) ***Mathematics/Arithmetic/Common Arithmetic Items/Percents.*** First, find the reduced price: $\$64-(25\%\text{ of }\$64)=\$64-(0.25\bullet\$64)=\$64-\$16=\$48$. Next, calculate the sales tax on \$48: $5\%\text{ of }\$48=0.05\bullet\$48=\$2.40$. Now, find the total cost: $\$48.00+\$2.40=\$50.40$.

42. (K) (p. 495) ***Mathematics/Algebra/Solving Simultaneous Equations.*** Use the method for solving simultaneous equations: $\frac{y}{z}=k-1\Rightarrow k=\frac{y}{z}+1$. Since $\frac{x}{z}=k$: $\frac{x}{z}=\frac{y}{z}+1\Rightarrow x=z\left(\frac{y}{z}+1\right)=y+z$.

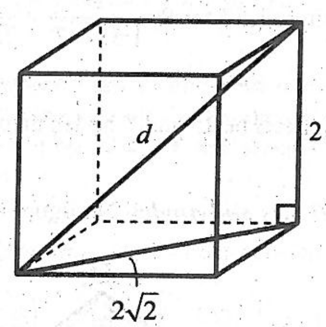
43. (A) (p. 495) ***Mathematics/Algebra/Solving Algebraic Equations with Two Variables.*** If $x=0.25y$, then $y=\frac{x}{0.25}=4x$. Thus, y is 400 percent of x.

44. (G) (p. 496) ***Mathematics/Arithmetic/Common Arithmetic Items/Properties of Numbers.*** Since 3 is a factor of 9 and 5 is a factor of 5, any multiple of both 9 and 5 will be a multiple of $3(5)=15$; (II) belongs in the correct choice. (I), however, is not correct. x could be any multiple of 45, e.g., 90, which also proves that (III) does not belong in the correct choice.

45. (B) (p. 496) ***Mathematics/Geometry/Complex Figures*** and ***Triangles/Pythagorean Theorem.*** The neat thing about a cube is that if given any one feature (e.g., volume, edge, diagonal of a face, diagonal of the cube, surface area), every other feature can be calculated. This is why cubes are often the focus of test problems. The edge has a length of 2, so use the Pythagorean theorem to find the length of the diagonal of a face:



$d^2=2^2+2^2=4+4=8\Rightarrow d=2\sqrt{2}$. Now, find the length of the diagonal of the cube:



$d^2=2^2+\left(2\sqrt{2}\right)^2=4+8=12\Rightarrow d=2\sqrt{3}$. That is the length of the entire diagonal of the cube. The point that is the center of the cube is the midpoint of the diagonal of the cube and is $\sqrt{3}$ units of length from each vertex.

46. (J) (p. 496) ***Mathematics/Geometry.*** $\text{Volume}_{\text{cylinder}} = \pi r^2 h$. Redefine the dimensions of the smaller cylinder in terms of r and h: $r = kr'$ so $r' = \frac{r}{k}$ and $h = kh'$ so $h' = \frac{h}{k}$. $\text{Volume}_{\text{smaller cylinder}} = \pi\left(\frac{r}{k}\right)^2\left(\frac{h}{k}\right) = \frac{\pi r^2 h}{k^3}$. The ratio is $\frac{\frac{\pi r^2 h}{k^3}}{\pi r^2 h} = \frac{1}{k^3}$. Therefore, the correct answer is (J), $1 : k^3$.

Alternatively, assume some numbers. Let the radius and height of the larger cylinder be 4 and 4, and those of the smaller cylinder be 2 and 2. Since $r = kr'$ and $h = kh'$, k must be 2. $\text{Volume}_{\text{larger cylinder}} = \pi(4)^2(4) = 64\pi$. $\text{Volume}_{\text{smaller cylinder}} = \pi(2)^2(2) = 8\pi$. The ratio 8π to 64π is 1 to 8 or $\frac{1}{8}$. Use $k = 2$ to find the answer that has a value of $\frac{1}{8}$:

F. $1 : \pi = \frac{1}{\pi}$ ✗

G. $\pi : 1 = \frac{\pi}{1} = \pi$ ✗

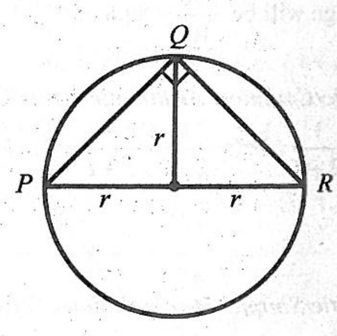
H. $k\pi : 1 = \frac{k\pi}{1} = 2\pi$ ✗

J. $1 : k^3 = \frac{1}{k^3} = \frac{1}{2^3} = \frac{1}{8}$ ✓

K. $k^3 : 1 = \frac{k^3}{1} = \frac{2^3}{1} = \frac{8}{1} = 8$ ✗

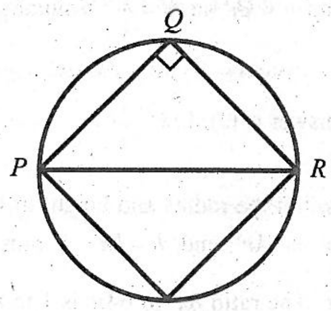
47. (A) (p. 497) ***Mathematics/Geometry/Complex Figures*** and ***Triangles/Pythagorean Theorem*** and ***Circles.*** This is a right triangle, so $\angle PQR$ intercepts an arc of 180°. (The inscribed angle $\angle PQR$ is equal to half its intercepted arc.) Because the arc is 180°, the hypotenuse of the triangle, $\overline{PR}$, is also the diameter of the circle. From any bit of information about a right isosceles triangle (e.g., either side lengths, the hypotenuse, or the area), the other information can be found. Using the two adjacent sides as the altitude and the base, we have: $\text{area}_{\text{triangle}} = \frac{1}{2}(s)(s) \Rightarrow 1 = \frac{1}{2}s^2 \Rightarrow s^2 = 2$. Now, use the Pythagorean theorem to solve for $\overline{PR}$: $s^2 + s^2 = \overline{PR}^2 \Rightarrow 2 + 2 = \overline{PR}^2 \Rightarrow 4 = \overline{PR}^2 \Rightarrow \overline{PR} = 2$. Since $\overline{PR} = 2$, the radius of the circle is 1, and $\text{area}_{\text{circle}} = \pi(1)^2 = \pi$.

The same conclusion can be arrived at in a slightly different manner:



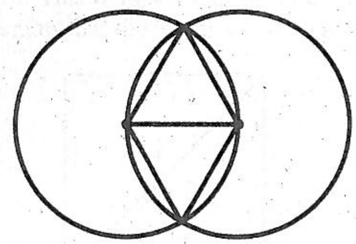
Based on the figure, r is the length of the altitude of the triangle and $2r$ is the length of the base, so $\text{area}_{\text{triangle}} = \frac{1}{2}(2r)(r) = r^2$ and $r^2 = 1 \Rightarrow r = 1$. Thus, $\text{area}_{\text{circle}} = \pi r^2 = \pi(1)^2 = \pi$.

Finally, a little common sense can solve this problem without any math. The triangle, which has an area of 1, takes up slightly less than half the circle:



The correct answer must be a bit larger than 2, and only one choice qualifies: (A) is π and π is slightly larger than 3. Therefore, (A) is reasonable. All of the other choices are more than 6 and so are too large to be reasonable.

48. **(F)** (p. 497) ***Mathematics/Geometry/Complex Figures*** and ***Circles.*** This is a good exercise in organized problem-solving. Look at the figure and ask what is known: the radius of the circle and the perimeter of the shaded area consists of two arcs. There must be some way to use the information about the radius to find the length of the arcs. Arcs can be measured in terms of length or in terms of degrees.



Since the sides of the triangles are all radii, the triangles must be equilateral, and the degree measure of each arc is 120. The circles have radii of 1, so they have circumferences of $2\pi(1) = 2\pi$. Each arc is a third of that length: $\frac{2\pi}{3}$. Since there are two such arcs, the perimeter of the shaded area is $2 \cdot \frac{2\pi}{3} = \frac{4\pi}{3}$.

49. **(C)** (p. 498) ***Mathematics/Statistics and Probability/Averages.*** For the first six tests, the student has a total point count of $6 \cdot 83 = 498$. If the student scores a 0 on each of the remaining four tests, the total point count will remain 498 and the average will be $\frac{498}{10} = 49.8$. If the student scores 100 on each of the four remaining tests, the total point count will be 898 and the average will be $\frac{898}{10} = 89.8$.

50. **(K)** (p. 498) ***Mathematics/Arithmetic/Common Arithmetic Items/Complex Numbers.*** Let x be the multiplicative inverse of $2-i$: $x(2-i)=1 \Rightarrow x=\frac{1}{2-i} \Rightarrow x=\frac{1}{2-i} \cdot \frac{2+i}{2+i}=\frac{2+i}{4-i^2}$. $i=\sqrt{-1}$, so $i^2=-1$. Thus, $x=\frac{2+i}{4-(-1)}=\frac{2+i}{5}$.

51. **(C)** (p. 498) ***Mathematics/Arithmetic/Simple Manipulations.*** From the exponential concept of logarithms, if $a^x = b$, then the equation can be written in logarithmic form as $\log_a b = x$. Let $\log_3 \sqrt{3} = x$; then $3^x = \sqrt{3} \Rightarrow 3^x = 3^{\left(\frac{1}{2}\right)}$. Thus, $x = \frac{1}{2}$.

52. (J) (p. 498) ***Mathematics/Algebra/Expressing and Evaluating Algebraic Functions/Function Notation.*** Study the structure of *f*. When will the function be at its minimum value? This is like asking for the minimum value of $(x-1)^2$. Squaring any positive number yields a positive number; squaring any negative number yields a positive number; and squaring zero yields zero. Since zero is less than any positive number, we want $x-1$ to equal zero, and this occurs when $x=1$. Now, plug in 1 for *x* in the given function and solve to find the minimum value of the function: $f(1)=(x-1)^2+2=(1-1)^2+2=2$.

Alternatively, test the answer choices. Plug each answer choice into the given function; the correct choice will be the lowest value:

F. $(-3-1)^2+2=(-4)^2+2=16+2=18$ ✗
G. $(-2-1)^2+2=(-3)^2+2=9+2=11$ ✗
H. $(0-1)^2+2=(-1)^2+2=1+2=3$ ✗
J. $(1-1)^2+2=(0)^2+2=0+2=2$ ✓
K. $(2-1)^2+2=(1)^2+2=1+2=3$ ✗

53. (A) (p. 499) ***Mathematics/Algebra/Manipulating Algebraic Expressions/Manipulating Expressions Involving Exponents.*** Simply use the rules for working with exponents to solve the given equation for *x*: $2^n+2^n+2^n+2^n=x(2^{n+1})\Rightarrow(4)(2^n)=x\left(2^{n+1}\right)\Rightarrow(2)\left(2^1\right)\left(2^n\right)=x\left(2^{n+1}\right)\Rightarrow 2\left(2^{n+1}\right)=x\left(2^{n+1}\right)\Rightarrow 2=x$.

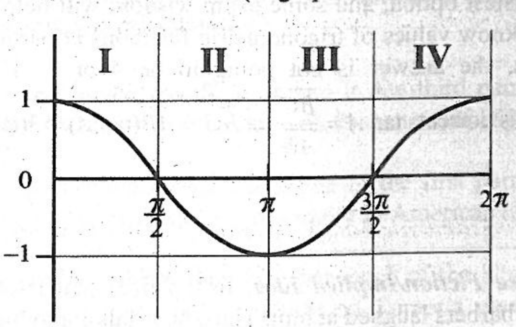
54. (F) (p. 499) ***Mathematics/Algebra/Expressing and Evaluating Algebraic Functions/Function Notation.*** $f(k)$ will equal $f(-k)$ when $(k)^2+2(k)+1=(-k)^2-2k+1\Rightarrow k^2+2k+1=k^2-2k+1\Rightarrow 2k=-2k\Rightarrow 4k=0\Rightarrow k=0$.

Another approach is to work backwards from the answer choices. First, use the value 0: $(0)^2+2(0)+1=(-0)^2-2(0)+1\Rightarrow 1=1$. Thus, 0 is part of the solution set; eliminate (G), (H), and (J). The only question that remains is whether the correct answer is (K). Take another value, say 1: $(1)^2+2(1)+1=(-1)^2-2(-1)^2+1=(-1)^2-2(1)+1\Rightarrow 4\neq 0$. This proves that the answer is (F).

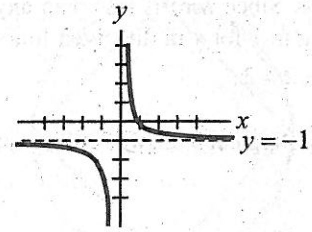
55. (D) (p. 499) ***Mathematics/Algebra/Manipulating Algebraic Expressions/Factoring Expressions.*** Factor the expression: $\frac{x^2-1}{x-1}=\frac{(x+1)(x-1)}{x-1}=x+1$. Therefore, as *x* approaches 1, $x+1$ approaches 2.

56. (H) (p. 499) ***Mathematics/Trigonometry/Determining Trigonometric Values.*** Given the restrictions on *x* and since $\cos x=-1$, *x* must equal π and $\cos\frac{\pi}{2}=0$.

Alternatively, visualize the graph of the cosine function (or graph it on a calculator):



57. **(B)** (p. 500) ***Mathematics/Algebra/Expressing and Evaluating Algebraic Functions/Concepts of Domain and Range.*** The domain of a function is the set of all possible x values; the range of a function is the set of all possible y values. Sketch a graph of the function (or use a graphing calculator):



From the graph, it is obvious that the y-values both approach, but never actually reach, -1. Therefore, the range is defined by all real numbers except -1.

Alternatively, this item can be solved algebraically by solving for x. Since $f(x)=\frac{1-x}{x}$, $x[f(x)]=1-x \Rightarrow x[f(x)]+x=1 \Rightarrow x[f(x)+1]=1 \Rightarrow x=\frac{1}{f(x)+1}$. The range of the function is the set of all possible values for $f(x)$. Since division by zero is undefined, $f(x)+1\neq 0 \Rightarrow f(x)\neq -1$. Thus, $f(x)$ can be any value except -1.

58. **(K)** (p. 500) ***Mathematics/Trigonometry/Trigonometric Relationships.*** $x=3(\sin\theta)$ and $y=2(\cos\theta)$; thus, $\sin\theta=\frac{x}{3}$ and $\cos\theta=\frac{y}{2}$. $\sin^2\theta+\cos^2\theta=1$; thus, $\left(\frac{x}{3}\right)^2+\left(\frac{y}{2}\right)^2=1 \Rightarrow \frac{x^2}{9}+\frac{y^2}{4}=1$ is the equation of an ellipse with center (0,0) that passes through the points (3,0) and (0,2).

Alternatively, plug in values for θ. For $\theta=0$, $x=3[\sin(0)]=3(0)=0$. For $\theta=\frac{\pi}{2}$, $x=3\left(\sin\frac{\pi}{2}\right)=3(1)=3$ and $y=2\left(\cos\frac{\pi}{2}\right)=2(0)=0$. Therefore, the graph must include the points (3,0) and (0,2). The only graph given that contains both of these points is (K).

59. **(A)** (p. 500) ***Mathematics/Trigonometry/Definitions of the Six Trigonometric Functions.*** Sine and cosecant are reciprocal functions, so the product of the sine of any angle and the cosecant of that angle is 1. This fact can quickly be derived from the definitions of sine and cosecant. Given a triangle with sides a and b and hypotenuse c, let θ be opposite side b. $\sin\theta=\frac{b}{c}$ and $\csc\theta=\frac{c}{b}$. Therefore, $\sin\theta\cdot\csc\theta=1$.

60. **(J)** (p. 501) ***Mathematics/Trigonometry/Trigonometry as an Alternative Method of Solution.*** There are several different ways of expressing the length of $\overline{BC}$: as a number, as a function of $\angle ACB$, and as a function of $\angle CAB$. "Testing-the-test" is the easiest option, and some exam wisdom will help. First, do not fall for (F) or (G). Test-takers are not expected to know values of trigonometric functions at particular angles. Since 40° and 50° are not easily remembered values, the answer is not going to be 4 or 5. The answer will be expressed using a trigonometric function. (J) is correct: $\tan A=\frac{\overline{BC}}{\overline{AB}} \Rightarrow \overline{BC}=\overline{AB}(\tan A)=3(\tan 50°)$.

Section 3: Reading

1. **(D)** (p. 503) ***Reading/Prose Fiction/Implied Idea.*** In the final paragraph, the young man, Robin, is trying to explain to himself why the barbers laughed at him. Thus, he is talking to himself.